MATH 4030 Problem Set 6[1]
Due date: No need to hand in

Reading assignment: do Carmo's Section 4.4, 4.5

Problems:

1. Show that if all the geodesics of a connected surface are plane curves, then the surface is contained in a plane or a sphere.

2. If $S \subset \mathbb{R}^3$ is a closed surface with positive Gauss curvature $K > 0$, show that any two simple closed geodesics on S must intersect.

3. Let $\Sigma \subset \mathbb{R}^3$ be a closed orientable surface which is not homeomorphic to a sphere. Prove that there are points on Σ where the Gauss curvature K is positive, negative and zero respectively.

4. Let T be a torus of revolution which can be parametrized (except along two curves) by

$$f(u,v) = ((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u), \qquad 0 < u < 2\pi, 0 < v < 2\pi.$$

   Prove by an explicit calculation that the total Gauss curvature is

$$\int_T K\, dA = 0.$$

   What are the geodesics on T?

5. Let p_0 be a pole of a unit sphere $\mathbb{S}^2$ and q, r be two points on the corresponding equator in such a way that the meridians p_0q and p_0r make an angle θ at p_0. Consider a unit vector v tangent to the meridian p_0q at p_0, and take the parallel transport of v along the closed curve made up by the meridian p_0q, the parallel qr, and the meridian rp_0 (Fig. 4-21). Determine the angle between the final position of v after parallel transport with the initial vector v.

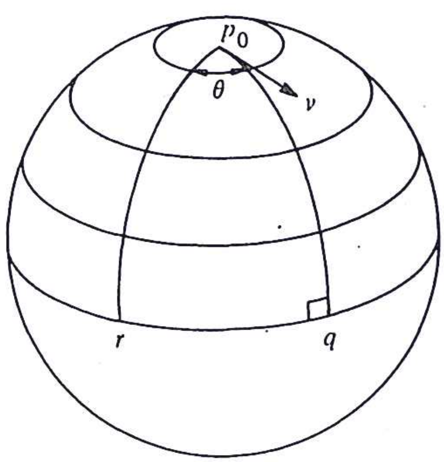

Figure 4-21

[1]Last revised on November 20, 2019